Research Article

M-Gonal Number ± 2 = A Perfect Square

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Abstract: **We search for the ranks of Triangular, Pentagonal, Heptagonal, Nanogonal and Tridecagonal numbers such that each of these M-gonal number - 2= a perfect square and the ranks of Pentagonal and Heptagonal such that each of these M-gonal number + 2= a perfect square.**

Keywords: **Triangular number, Pentagonal number, Heptagonal number, Nanogonal number and Tridecagonal number**

I. INTRODUCTION

Number is the essence of mathematical calculation. Variety of numbers have variety of range and richness, Many numbers exhibit fascinating properties, they form sequences, they form patterns and so on [1]. In [3] explicit formulas for the ranks of triangular numbers which are equal to Pentagonal, Octagonal, Decagonal and Dodecagonal numbers in turn are presented. In this context one may refer [4-8]. In this communication, we make an attempt to obtain the ranks of Triangular, Pentagonal, Heptagonal, Nanogonal and Tridecagonal numbers such that each of these M-gonal number - 2 = a perfect square and the ranks of Pentagonal and Heptagonal such that each of these M-gonal number + 2= a perfect square. Also the recurrence relations satisfied by the ranks of each of these M-gonal numbers are presented.

II. METHOD OF ANALYSIS

Pattern 1:

Denoting the rank of the n^{th} Triangular number to be A, the identity,

Triangular number – 2 = x^2 (1)

is written as

$$y^2 = 8x^2 + 17 \quad (2)$$

Where $y = 2A + 1$ (3)

whose initial solution is $x_0 = 1, y_0 = 5$ (4)

Let $(\tilde{x}_0, \tilde{y}_0)$ be the general solution of the Pellian

$$y^2 = 8x^2 + 1 \quad (5)$$

where $\tilde{x}_s = \frac{1}{2\sqrt{8}}\left(\left(3+\sqrt{8}\right)^{s+1} - \left(3-\sqrt{8}\right)^{s+1}\right)$

$$\tilde{y}_s = \frac{1}{2}\left(\left(3+\sqrt{8}\right)^{s+1} + \left(3-\sqrt{8}\right)^{s+1}\right), \quad s = 0,1,....$$

Applying Brahmagupta's lemma [2] between the solutions (x_0, y_0) and $(\tilde{x}_s, \tilde{y}_s)$ the sequence of values of x and y satisfying equation (2) is given by

$$x_s = \frac{1}{2\sqrt{8}}\left(\left(3+\sqrt{8}\right)^{s+1}\left(5+\sqrt{8}\right) - \left(3-\sqrt{8}\right)^{s+1}\left(5-\sqrt{8}\right)\right)$$

$$y_s = \frac{1}{2}\left(\left(3+\sqrt{8}\right)^{s+1}\left(5+\sqrt{8}\right) + \left(3-\sqrt{8}\right)^{s+1}\left(5-\sqrt{8}\right)\right), \quad s = 0,1,....$$

Inview of (3), the ranks of Triangular number is given by

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$$A_{2s-1}=\frac{1}{4}\left(\left(3+\sqrt{8}\right)^{2s}\left(5+\sqrt{8}\right)+\left(3-\sqrt{8}\right)^{2s}\left(5-\sqrt{8}\right)-2\right),\ s=1,2...$$

and the corresponding recurrence relation is found to be

$$A_{2s+3}=34A_{2s+1}-A_{2s-1}+16$$

In a similar manner, we present below the ranks of Pentagonal, Heptagonal, Nanogonal and Tridecagonal numbers in tabular form

S.No	M-Gonal number	General form of ranks
1	Pentagonal number (B)	$B_{2s-1}=\frac{1}{12}\left(\left(5+\sqrt{24}\right)^{2s}\left(35+7\sqrt{24}\right)+\left(5-\sqrt{24}\right)^{2s}\left(35-7\sqrt{24}\right)+2\right),\ s=1,2...$
2	Heptagonal number (C)	$C_{2s-1}=\frac{1}{20}\left(\left(19+3\sqrt{40}\right)^{2s}\left(27+4\sqrt{40}\right)+\left(19-3\sqrt{40}\right)^{2s}\left(27-4\sqrt{40}\right)+6\right)$, $s=1,2...$
3	Nanogonal number (D)	$D_{2s-1}=\frac{1}{56}\left(\left(15+\sqrt{224}\right)^{2s}\left(37+2\sqrt{224}\right)+\left(15-\sqrt{224}\right)^{2s}\left(37-2\sqrt{224}\right)+10\right)$ $s=1,2...$
4	Tridecagonal number (E)	$E_{2s-1}=\frac{1}{44}\left(\left(197+21\sqrt{88}\right)^{2s}\left(123+13\sqrt{88}\right)+\left(197-21\sqrt{88}\right)^{2s}\left(123-21\sqrt{88}\right)+18\right)$ $s=1,2...$

The recurrence relations satisfied by the ranks of each the M-Gonal numbers presented in the table above are as follows

S.NO	RECURRENCE RELATIONS
1	$B_{2s+3}=98B_{2s+1}-B_{2s-1}-16$
2	$C_{2s+3}=144C_{2s+1}-C_{2s-1}-432$
3	$D_{2s+3}=898D_{2s+1}-D_{2s-1}-160$
4	$E_{2s+3}=155234E_{2s+1}-E_{2s-1}-63504$

Pattern 2:

Denoting the rank of the nth Pentagonal number to be B, the identity,

Pentagonal number + 2 = x^2 (6)

is written as

$y^2=24x^2-47$ (7)

Where $y=6B-1$ (8)

whose initial solution is $x_0=1, y_0=5$ (9)

Let $(\tilde{x}_0,\tilde{y}_0)$ be the general solution of the Pellian

$y^2=24x^2+1$ (10)

where $\tilde{x}_s=\frac{1}{2\sqrt{24}}\left(\left(5+\sqrt{24}\right)^{s+1}-\left(5-\sqrt{24}\right)^{s+1}\right)$

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$$\tilde{y}_s = \frac{1}{2}\left(\left(5+\sqrt{24}\right)^{s+1} + \left(5-\sqrt{24}\right)^{s+1}\right), \quad s = 0,1,....$$

Applying Brahmagupta's lemma [2] between the solutions (x_0, y_0) and $(\tilde{x}_s, \tilde{y}_s)$ the sequence of values of x and y satisfying equation (7) is given by

$$x_s = \frac{1}{2\sqrt{24}}\left(\left(5+\sqrt{24}\right)^{s+1}\left(7+2\sqrt{24}\right) - \left(5-\sqrt{24}\right)^{s+1}\left(7-2\sqrt{24}\right)\right)$$

$$y_s = \frac{1}{2}\left(\left(5+\sqrt{24}\right)^{s+1}\left(7+2\sqrt{24}\right) + \left(5-\sqrt{24}\right)^{s+1}\left(7-\sqrt{24}\right)\right), \quad s = 0,1,....$$

Inview of (8), the ranks of Pentagonal number is given by

$$B_{2s-1} = \frac{1}{12}\left(\left(5+\sqrt{24}\right)^{2s}\left(7+2\sqrt{24}\right) + \left(5-\sqrt{24}\right)^{2s}\left(7-2\sqrt{24}\right) + 2\right), \quad s = 1,2...$$

and the corresponding recurrence relation is found to be

$$B_{2s+3} = 98B_{2s+1} - B_{2s-1} - 16$$

On following the procedure similar to the above for Heptagonal number, we get the rank and recurrence relation which are given below

$$C_{2s-1} = \frac{1}{20}\left(\left(19+3\sqrt{40}\right)^{2s}\left(17+3\sqrt{40}\right) + \left(19-3\sqrt{40}\right)^{2s}\left(17-3\sqrt{40}\right) + 6\right), \quad s = 1,2...$$

$$C_{2s+3} = 144C_{2s+1} - C_{2s-1} - 432$$

III. CONCLUSION

To conclude, one may search for the other M-gonal numbers satisfying the relation under consideration.

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